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# Ordinary Differential Equations / Dynamical Systems

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**ECTS**: 6 ECTS.

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**COORDINATOR**: Óscar López Pouso (oscar.lopez@usc.es).

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**UNIVERSITY WHERE THE COORDINATOR IS:** USC.

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**HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES?** Yes, I have.

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**LECTURER 1:** Jerónimo Rodríguez García (jeronimo.rodriguez@usc.es).

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**UNIVERSITY WHERE THE LECTURER 1 IS:** USC.

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**HAVE YOU GIVEN PERMISSION TO RECORD YOUR CLASSES?** Yes, I have.

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**SUBJECT CONTENTS**

I. NUMERICAL METHODS FOR INITIAL VALUE PROBLEMS ASSOCIATED TO ORDINARY DIFFERENTIAL EQUATIONS (ODEs):

1. Concept of initial value problem (IVP) for ODEs. Idea of numerical solution of an IVP.
2. MATLAB® commands for solving IVPs.
3. Definition of convergence and order of convergence. Discretization error and rounding error; effect of rounding errors on convergence.
4. Description of Euler methods: explicit (forward) and implicit (backward).
5. High order methods:
    a. One-step non-linear methods: Runge-Kutta (RK) methods.
    b. Linear multistep methods (LMM):
        i. Concept of LMM. Starting procedure. Order theorem.
        ii. LMM based on numerical integration:
            - Adams-Bashforth methods.

- Adams-Moulton methods.
- Nyström methods.
- Milne-Simpson methods.

iii. LMM based on numerical differentiation: BDF methods.

II. DYNAMICAL SYSTEMS:

1. Linear dynamical systems.

   a. Linear vector fields.
   b. Calculus of the exponential of a matrix. Jordan canonical form.
   c. Fundamental theorem of existence and uniqueness of solution for linear systems.
   d. Invariant subspaces: stable, unstable and central spaces.

2. Basic theorems related to the general theory of differential equations.

   a. The fundamental theorem of existence and uniqueness of solution. Dependence on the parameters and on the initial conditions.
   b. The problem of extension of solutions. Maximal solutions
   c. Flux associated to a differential field. Singular and regular points. Orbits. α-limit and ω-limit sets.

3. Local theory.

    a. Lyapunov stability. Lyapunov functions.
    b. Concepts of equivalence and topological conjugacy. Structural stability.
    c. The invariant manifold theorem.
    d. The Hartman-Grobman theorem.
    e. Gradient and Hamiltonian systems.

4. Global theory.

    a. The concept of limit cycle.
    b. Electric circuits. Liénard systems. The Van der Pol equation.
    c. The Poincaré map.

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**METHODOLOGY**

1. Planning for the contents of each class.
2. Explanation on blackboard (lecture) or equivalent by using videoconferencing.
3. Programming some methods on the computer.

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**LANGUAGE USED IN CLASS:** Spanish.

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**IS IT COMPULSORY TO ATTEND CLASS?** It is not compulsory.

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## BIBLIOGRAPHY

### I. NUMERICAL METHODS FOR INITIAL VALUE PROBLEMS (IVPs) ASSOCIATED TO ORDINARY DIFFERENTIAL EQUATIONS (ODEs):

BASIC BIBLIOGRAPHY:

1. ASCHER, URI M.; PETZOLD, LINDA R. (1998) Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations. SIAM, Philadelphia, PA.
2. HAIRER, ERNST; NØRSETT, SYVERT PAUL; WANNER, GERHARD (1987) Solving Ordinary Differential Equations I. Nonstiff Problems. Springer, Berlin.
3. ISAACSON, EUGENE; KELLER, HERBERT BISHOP (1994, unabridged, corrected republication) Analysis of Numerical Methods. Dover Publications, New York, NY. [Original edition: Wiley, 1966].
4. ISERLES, ARIEH (2008, second edition) A first course in the numerical analaysis of differential equations. Cambridge Texts in Applied Mathematics. Cambridge University Press. Cambridge. [First edition: 1997.]
5. LAMBERT, JOHN DENHOLM (1991) Numerical Methods for Ordinary Differential Systems. Wiley, Chichester.
6. STOER, JOSEF; BULIRSCH, ROLAND (1993, second edition) Introduction to Numerical Analysis. Springer, New York, NY. [First edition: 1980].

COMPLEMENTARY BIBLIOGRAPHY:

1. BUTCHER, JOHN CHARLES (2008, second edition) Numerical Methods for Ordinary Differential Equations. Wiley, Chichester. [First edition: 2003.]
2. CROUZEIX, MICHEL; MIGNOT, ALAIN L. (1989, second edition) Analyse Numérique des Équations Différentielles. Masson, Paris. [First edition: 1984.]
3. DEKKER, KEES; VERWER, JAN G. (1984) Stability of Runge-Kutta Methods for Stiff Nonlinear Differential Equations. Elsevier Science Publishers B. V., Amsterdam.
4. HAIRER, ERNST; WANNER, GERHARD (1991) Solving Ordinary Differential Equations II. Stiff and Differential-Algebraic Problems. Springer, Berlin.
5. HENRICI, PETER (1962) Discrete Variable Methods in Ordinary Differential Equations. Wiley. New York, NY.
6. KINCAID, DAVID RONALD; CHENEY, ELLIOT WARD (1991) Numerical Analysis. Brooks/Cole, Pacific Grove, CA.
7. LAMBERT, JOHN DENHOLM (1973) Computational Methods in Ordinary Differential Equations. Wiley, London.
8. QUARTERONI, ALFIO; SACCO, RICCARDO; SALERI, FAUSTO (2000) Numerical Mathematics. Springer, New York, NY.

### II. DYNAMICAL SYSTEMS:

BASIC BIBLIOGRAPHY:

1. PERKO, LAWRENCE (2000, third edition). Differential Equations and Dynamical Systems. Texts in Applied Mathematics 7. Springer.
2. HIRSCH, MORRIS W.; SMALE, STEPHEN (1974). Differential Equations, Dynamical Systems and Linear Algebra. Pure and Applied Mathematics. Academic Press.

COMPLEMENTARY BIBLIOGRAPHY:

1. GUCKENHEIMER, JOHN; HOLMES, PHILIP (1983). Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Springer-Verlag New York.

2. HALE, JACK K.; KOÇAK, HÜSEYIN (1991). Dynamics and Bifurcations. Springer-Verlag New York.

3. HAIRER, ERNST; NØRSETT, SYVERT PAUL; WANNER, GERHARD (1987) Solving Ordinary Differential Equations I. Nonstiff Problems. Springer, Berlin.

## SKILLS

Basic and general skills:

CG1: To have knowledge that provide a basis or opportunity for originality in developing and / or applying ideas, often within a research context, knowing how to translate industrial needs in terms of R & D in the field of mathematics Industrial.

CG4: To have the ability to communicate the findings to specialist and non-specialist audiences in a clear and unambiguous way.

CG5: To have the appropriate learning skills to enable them to continue studying in a way that will be largely self-directed or autonomous, and also to be able to successfully undertake doctoral studies.

Specific skills:

CE3: To determine if a model of a process is well made and well mathematically formulated from a physical standpoint.

Skills of "Modelling" specialization:

CM1: To be able to extract, using different analytical techniques, both qualitative and quantitative models.

The student receives training on the above mentioned skills, as well as the ones described on page 8 in the official documentation of the Master at

http://www.usc.es/export/sites/default/gl/servizos/sxopra/memorias_master_USC/P4151_Master_Matematica_Industrial_memoria_def.pdf,

The assessment is made following the specifications that can be found in the section "Criteria for assessment...".

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**WILL YOU BE USING A VIRTUAL PLATFORM?** Yes, we will. Moodle (USC)

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**WILL YOU BE USING ANY SPECIFIC SOFTWARE?** Yes, we will. MATLAB and MAPLE.

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## CRITERIA FOR THE 1ST ASSESSMENT OPPORTUNITY

Skills CG1, CG4 and CG5, as well as CE3 and CM1, are assessed by means of the following process:

To pass the course it is compulsory to hand in the exercises and programming practices commissioned by the teachers within the timeframes set to it. The final grade will result from a written examination in which:

- Each of the two parts of this subject, namely Numerical Methods for ODEs on the one hand and Dynamical Systems on the other hand, weighs the 50% of the total mark.
- The part of the exam devoted to Numerical Methods for ODEs reserves 30% of its value for questions related to the programming practices.

To attend or not to attend classes will not have influence in the final mark.

## CRITERIA FOR THE 2ND ASSESSMENT OPPORTUNITY

The same criteria as the ones used in the first assessment opportunity.

## FURTHER COMMENTS:

Teachers are willing to teach in English. As of today, this can be done only in case every student accept the change.

The order in which the two parts of this subject are explained, namely Numerical Methods for ODEs on the one hand and Dynamical Systems on the other hand, will be communicated to the students at the beginning of each academic course.